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CHART FOR SIMPLIFYING CALCULATIONS OF PRESSURE DROP OF A HIGH-SPEED COMPRESSIBLE FLUID UNDER SIMULTANEOUS ACTION OF FRICTION AND HEAT TRANSFER - APPLICATION TO COMBUSTION-CHAMBER COOLING PASSAGES

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SUMMARY

A method for calculating the pressure drop of a high-speed compressible fluid in a constant-area duct under the simultaneous action of friction and heat transfer is developed. The method is based on the assumption of an exponential longitudinal temperature distribution. It is shown that the temperature distributions found in combustion-chamber cooling passages can be approximated by the assumed temperature distribution, and that a working chart based on this method simplifies the calculation of pressure drops across these passages. An illustrative example is included.

INTRODUCTION

An investigation of the cooling of ram-jet and tail-pipe-burner combustion chambers by means of air flowing through an annular cooling passage is being conducted at the NACA Lewis laboratory. The design of a combustion-chamber cooling system includes the calculation of the mass flow of cooling air necessary to maintain permissible wall temperatures and the determination of the pressure drop across the cooling passage required to obtain the necessary mass flow of cooling air.

The pressure drop in a cooling passage results from the simultaneous action of friction and heat transfer. For low rates of heat transfer and a Mach number below 0.5, the pressure drop may be calculated by the simplified methods of references 1 to 3; these methods result in appreciable errors at higher Mach numbers. Accurate calculation of the pressure drop requires solution of a basic differential equation describing the variation of pressure of a compressible fluid under the simultaneous action of friction and heat transfer. Unfortunately, this equation is not generally amenable to formal integration. Tables and charts that reduce the labor of numerical solution are given in references 4 and 5 and a special solution (reference 6) has been developed for the case of constant wall temperature.

For one-dimensional flow, the pressure at a point is uniquely defined when the corresponding values of mass flow, temperature, Mach number, and flow area are known. The pressure change between two points along a constant-area duct can be determined if the temperatures and Mach numbers at the two points are known. The differential equation relating Mach number variations and temperature distribution can be integrated in special cases. One of these cases occurs where the longitudinal temperature distribution T is given by $T = ce^{nx}$ (where c and n are constants and x is the distance along the passage). It is shown herein that the longitudinal distribution of air temperature in the cooling passages of ram-jet and tail-pipe-burner combustion chambers can be closely approximated by the use of the foregoing equation in a series of steps, each step having different values for the constants c and n. This analysis develops a working chart that simplifies the determination of the Mach number change in a combustion-chamber cooling passage and, consequently, simplifies the calculation of pressure drops required across these passages to obtain the necessary mass flow of cooling air. The method developed applies to the flow of air through straight ducts having any constant shape, cross-sectional area, and roughness.

The solution is presented in the form of a working chart constructed for a ratio of specific heats equal to 1.40. The variables cover ranges sufficiently large to include almost all applications of engineering interest. An example illustrating the use of the chart for a typical problem is presented.

SYMBOLS

The following symbols are used in this report:

A flow area, square feet

$$B = \frac{M^2\left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{\gamma+1}{\gamma+1+2\gamma K}}}{\left[1+\gamma(1+K)M^2\right]^{\frac{2(\gamma+1+\gamma K)}{\gamma+1+2\gamma K}}}$$

1238

B^* value of B when $M = 1.0$

D_h hydraulic diameter, $\frac{4 \times \text{flow area}}{\text{wetted perimeter}}$, feet

F friction drag, pounds

f friction factor as defined by, $\frac{F}{A} = 4f \frac{\rho V^2}{2} \frac{l}{D_h}$

$\bar{f}$ effective over-all value of friction factor f

G mass velocity of fluid, pounds per second per square foot

g acceleration of gravity, feet per second per second

$K = \dfrac{4\bar{f}l/D_h}{\log_e T_2/T_1}$

L^* distance from any given point to point where Mach number would theoretically equal 1.0

l length of passage, feet

M Mach number

P total pressure, pounds per square foot absolute

p static pressure, pounds per square foot absolute

R gas constant, foot-pounds per pound $^{\circ}R$

T total temperature, $^{\circ}R$

t static temperature, $^{\circ}R$

V fluid velocity, feet per second

x distance along passage, feet

γ ratio of specific heats

ρ mass density, slugs per cubic foot

Subscripts:

1 inlet 1238

2 outlet

METHOD OF ANALYSIS

For one-dimensional flow, the pressure at any point in a fluid is uniquely defined by the mass velocity, the temperature, and the Mach number at that point. The following development deals with the variation of Mach number along a duct. If the variation of Mach number is known, the corresponding pressure distribution is readily determined from the continuity equation for the one-dimensional flow of a perfect fluid as expressed by

$$\frac{G\sqrt{t}}{p} = M\sqrt{\frac{\gamma g}{R}} \tag{1}$$

or in terms of total temperature and total pressure

$$\frac{G\sqrt{T}}{P} = M\sqrt{\frac{\gamma g}{R}}\left(1 + \frac{\gamma-1}{2}M^2\right)^{-\frac{\gamma+1}{2(\gamma-1)}} \tag{1a}$$

Equation (1a) is graphically presented in figure 1.

One form of the differential equation for the variation of Mach number of a compressible fluid under the simultaneous action of friction and heat transfer for constant area and specific heat is obtained from table 2 of reference 7.

$$\frac{dM^2}{M^2} = \frac{1+\gamma M^2\left(1 + \frac{\gamma-1}{2}M^2\right)}{1-M^2}\frac{dT}{T} + \frac{\gamma M^2\left(1 + \frac{\gamma-1}{2}M^2\right)}{1-M^2}\frac{4fdx}{D_h} \tag{2}$$

The change in Mach number is dependent on the temperature distribution along the duct. One of the cases for which equation (2) is integrable occurs when the longitudinal temperature distribution is given by

$$\frac{T}{T_1} = e^{\frac{4\bar{f}\,x}{D_h K}} \qquad (3)$$

1238 where K is an arbitrary constant (fig. 2). Then by differentiation

$$\frac{4\bar{f}dx}{D_h K} = \frac{dT}{T} \qquad (3a)$$

If in equation (2) f equals $\bar{f}$, the substitution of equation (3a) into equation (2) gives

$$\frac{dM^2}{M^2} = \frac{\left(1 + \frac{\gamma-1}{2} M^2\right)\left[1 + \gamma M^2 (1 + K)\right]}{1-M^2}\,\frac{dT}{T} \qquad (4)$$

Integration of equation (4)(appendix A) gives

$$\frac{T_2}{T_1} = \frac{\dfrac{M_2{}^2\left(1 + \frac{\gamma-1}{2} M_2{}^2\right)^{\frac{\gamma+1}{\gamma+1+2\gamma K}}}{\left[1+\gamma(1+K)M_2{}^2\right]^{\frac{2(\gamma+1+\gamma K)}{\gamma+1+2\gamma K}}}}{\dfrac{M_1{}^2\left(1 + \frac{\gamma-1}{2} M_1{}^2\right)^{\frac{\gamma+1}{\gamma+1+2\gamma K}}}{\left[1+\gamma(1+K)M_1{}^2\right]^{\frac{2(\gamma+1+\gamma K)}{\gamma+1+2\gamma K}}}} = \frac{B_2}{B_1} \qquad (5)$$

When $x = l$, $T = T_2$; then from equation (3)

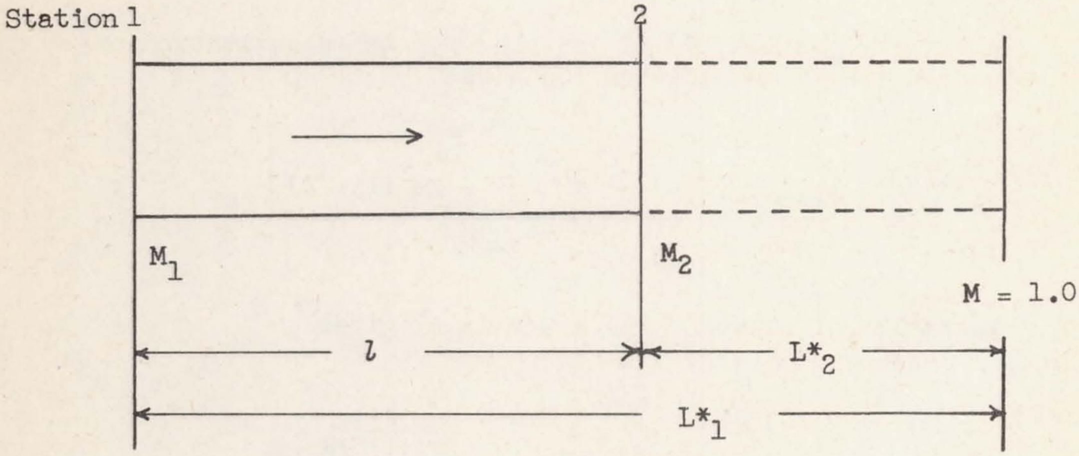
$$\frac{4\bar{f}l}{KD_h} = \log_e \frac{T_2}{T_1} = \log_e \frac{B_2}{B_1} \qquad (5a)$$

and

$$\frac{4\bar{f}L^*}{KD_h} = \log_e \frac{B^*}{B_1} \qquad (5b)$$

where L^* is the distance from any given point to the point where the Mach number would theoretically equal 1.0. Equation (5b) is graphically presented in the chart (fig. 3) for various values of K and for $\gamma=1.40$. (A 16 by 16 inch working chart of this figure is enclosed.) When $K=\pm\infty$, by equation (3) $T_2/T_1 = 1$, which reduces the problem to the case of isothermal flow with friction. 1238

The use of the choking-length parameter $4\bar{f}L^*/D_h$ is illustrated in the following example:



Given are $M_1 = 0.200$, $K = 1.0$, $l = 2.0$ feet, and $4\bar{f}/D_h = 0.5$. From figure 3,

$$\frac{4\bar{f}L^*_1}{D_h} = 1.5$$

$$L^*_1 = 3.0 \text{ feet}$$

By subtraction

$$L^*_2 = L^*_1 - l = 3.0 - 2.0 = 1.0 \text{ foot}$$

and

$$\frac{4\bar{f}L^*_2}{D_h} = 0.5$$

Then, from figure 3,

$$M_2 = 0.398$$

For $\gamma = 1.350$, the values of $4\bar{f}L^*/D_h$ are lower with a maximum difference of 5 percent; for $\gamma = 1.30$, the maximum difference would be 10 percent.

If the Mach number along a duct is constant, according to its definition L^* becomes infinite; if the Mach number along a duct decreases, its value never reaches 1.0 and L^* has no positive value. Consequently, the results of this analysis can be applied only to flow systems in which the Mach number increases.

For any initial Mach number, there is a critical value of K for which the Mach number along the duct is constant. This value can be determined by setting equation (4) equal to zero. The critical value of K is therefore

$$K = -\frac{1+\gamma M^2}{\gamma M^2} \tag{6}$$

Consequently, for a given Mach number, if the value of the friction factor is known, the minimum value of T_2/T_1 for which the Mach number increases, and therefore for which the chart can be applied, can be determined from equations (6) and (3).

APPLICATION TO COMBUSTION-CHAMBER COOLING PASSAGES

An annular cooling passage formed by a concentric liner inside a tail-pipe burner or ram-jet combustion chamber is shown in figure 4. If the rise in combustion-gas total temperature is approximately linear, the longitudinal temperature distribution of the cooling air may be similar to the distributions shown in figure 5. If the initial cooling-air and combustion-gas temperatures are equal and no heat losses occur through the outer wall of the cooling passage, the temperature distributions will be similar to those shown for case A (fig. 5(a)). Such a case would be an insulated tail-pipe burner in which a portion of the turbine-discharge gases are used as the coolant. If the initial cooling-air and combustion-gas temperatures are equal, but the external heat losses are large, the temperature distributions will be similar to those shown for case B (fig. 5(b)). This case would correspond to an uninsulated tail-pipe burner in which the outside wall of the cooling annulus is cooled by ambient air. Any one of the temperature distributions of case A in figure 5(a) can be approximated (case A, fig. 6(a)) by equation (3), where K is determined by letting $T = T_2$ and $x = l$. The temperature distributions for case A in figure 5(a), however, have an initial slope of zero because the initial cooling-air and combustion-gas temperatures were equal. If, at the cooling-passage entrance, the cooling air has a lower temperature than the combustion gas, the cooling-air temperature distribution will have a positive initial slope. Inasmuch as the assumed temperature distribution (given by equation (3)) has a positive initial slope, it is noted that for two cases (zero and positive initial slopes) with the same entrance Mach number and over-all temperature ratio, the case with a positive initial slope can be more closely approximated in one step.

Any of the distributions for case A in figure 5(a) can be matched more closely by a two-step approximation if the temperature at some intermediate station (preferably near the midpoint) is known. In this case a separate value of K is determined for each step, as illustrated in appendix B. Similarly, the temperature distributions for case B in figure 5(b) can be approached in two steps (fig. 6(b)). In this case an intermediate value of x near the minimum cooling-gas temperature would be preferable; the value of K for the first step would then be negative.

The variation of Mach number (fig. 7) along the cooling passage for both original and approximate temperature distributions shown in figure 6 were calculated by a step-by-step integration of equation (2) using the tables of reference 4. A sufficient number of steps were taken to insure that any increase in the number of steps would not affect the result. In case A (fig. 7(a)), the important effect of small changes in the initial Mach number is shown. When the calculations show that sonic velocity is attained before the outlet of the cooling passage is reached, the chosen value of initial Mach number is greater than is physically possible.

Case A (fig. 7(a)) shows that the Mach number increases from 0.5 to 1.0 in a very short distance. In spite of the fact that this analysis is exact for the temperature distribution of equation (3), and can be made to closely approximate temperature distributions similar to those in figure 5, in a physical case small inaccuracies in the values chosen for $\overline{f}$ or D_h may introduce appreciable errors in the region of rapidly increasing Mach number. For example, if the length of the duct in figure 7(a) were 2.95 feet (fig. 8), the outlet Mach number based on the original temperature profile would be 1.0; if the value of $\overline{f}/D_h$ was then reduced by 10 percent, the outlet Mach number would be reduced to 0.75. At the point where the original value of $\overline{f}/D_h$ gives $M = 0.50$, however, a 10-percent reduction in the value of $\overline{f}/D_h$ would only reduce the Mach number to 0.46. These differences in outlet Mach number would give proportionate discrepancies in outlet static pressure and smaller discrepancies in outlet total pressure. Due to the uncertainty in choosing a friction factor or in determining the exact cross-sectional area for any physical case, values of outlet static pressure based on an outlet Mach number between 1.0 and approximately 0.5 may therefore be subject to appreciable error. Equations for estimating the value of the friction factor are given in reference 8. The effect of the ratio of the inner to the outer diameter on the friction factor of an annulus is discussed in reference 9.

The values of outlet Mach number obtained by the step-by-step integration of equation (2) along a variety of temperature-distribution curves similar to those of figure 5 were compared with the values of outlet Mach number obtained by the use of figure 3 (in a manner similar to that shown in appendix B), with the following results:

(1) When neither method indicated choking, the outlet Mach number based on a one-step approximation was 0 to 25 percent greater than the value obtained by step-by-step integration, with the greater differences occurring at the higher values of outlet Mach number. When both methods caused choking, the theoretical distance to choke based on a one-step approximation was 7 to 25 percent less than the distance obtained by step-by-step integration. Inasmuch as the temperature along these one-step-approximate curves was appreciably higher than the corresponding temperature on the original curves, this result was anticipated.

(2) For a two-step approximation, the outlet Mach numbers (unchoked by either method) were within 2 percent; the difference in theoretical distance to cause choking (choked by either method) was within 2 percent.

(3) For any value of outlet Mach number, the precentage change in outlet static or total pressure is equal to or less than the Mach number discrepancies mentioned in results 1 and 2, but in the opposite sense.

CONCLUDING REMARKS

A method is developed for calculating the pressure drop of a compressible fluid under the simultaneous action of friction and heat transfer, and is presented in the form of a working chart. It is shown that the temperature distribution assumed in the development of the working chart can be made to closely approximate, in one or two steps, the temperature distributions found in the cooling passages of ram-jet and tail-pipe-burner combustion chambers. When two steps were used, the outlet pressure was within 2 percent of the values that would be obtained by a step-by-step integration along the actual temperature-distribution curve.

Lewis Flight Propulsion Laboratory,
National Advisory Committee for Aeronautics,
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1238

APPENDIX A

INTEGRATION-OF-FLOW EQUATION

One form of the differential equation for simultaneous friction and heat transfer (reference 7) for constant area and specific heat is (equation (2) of text)

$$\frac{dM^2}{M^2} = \frac{1+\gamma M^2\left(1 + \frac{\gamma-1}{2} M^2\right)}{1-M^2} \frac{dT}{T} + \frac{\gamma M^2\left(1 + \frac{\gamma-1}{2} M^2\right)}{1-M^2} \frac{4fdx}{D_h} \tag{A1}$$

If the fluid temperature distribution is given by

$$\frac{dT}{T} = \frac{4fdx}{D_h K} \tag{A2}$$

substitution of equation (A2) in equation (A1) and factoring gives

$$\frac{dT}{T} = \frac{(1-M^2) \quad dM^2}{M^2\left(1 + \frac{\gamma-1}{2} M^2\right)\left[1+\gamma(1+K)M^2\right]} \tag{A3}$$

Let $a = \frac{\gamma-1}{2}$ and $b = \gamma(1+K)$. Equation (A3) can then be written as

$$\frac{dT}{T} = \frac{(1-M^2)dM^2}{M^2(1+aM^2)(1+bM^2)} \tag{A3a}$$

or by separating into partial fractions, equation (A3a) becomes

$$\frac{dT}{T} = \frac{(1-M^2)dM^2}{M^2} + \frac{a^2}{b-a} \frac{(1-M^2)dM^2}{(1 + aM^2)} + \frac{b^2}{a-b} \frac{(1-M^2)dM^2}{(1+bM^2)} \tag{A4}$$

Integration of equation (A4) results in

$$\int_1^2 \frac{dT}{T} = \int_1^2 \frac{dM^2}{M^2} - \int_1^2 dM^2 + \frac{a^2}{b-a}\left(\int_1^2 \frac{dM^2}{1+aM^2} - \int_1^2 \frac{M^2dM^2}{1+aM^2}\right) + \frac{b^2}{a-b}\left(\int_1^2 \frac{dM^2}{1+bM^2} - \int_1^2 \frac{M^2dM^2}{1+bM^2}\right) \tag{A5}$$

$$\log_e T\Bigg]_1^2 = \log_e M^2\Bigg]_1^2 - M^2\Bigg]_1^2 + \frac{a^2}{b-a}\left\{\frac{1}{a}\log_e(1+aM^2) - \frac{1}{a^2}\left[1+aM^2 - \log_e(1+aM^2)\right]\right\}\Bigg]_1^2 +$$

$$\frac{b^2}{a-b}\left\{\frac{1}{b}\log_e(1+bM^2) - \frac{1}{b^2}\left[1+bM^2 - \log_e(1+bM^2)\right]\right\}\Bigg]_1^2 \tag{A6}$$

$$\log_e T\Bigg]_1^2 = \log_e M^2\Bigg]_1^2 - M^2\Bigg]_1^2 + \frac{a+1}{b-a}\log_e(1+aM^2)\Bigg]_1^2 - \frac{1+aM^2}{b-a}\Bigg]_1^2 - \frac{b+1}{b-a}\log_e(1+bM^2)\Bigg]_1^2 + \frac{1+bM^2}{b-a}\Bigg]_1^2 \tag{A7}$$

However,

$$-M^2 - \frac{1+aM^2}{b-a} + \frac{1+bM^2}{b-a} = \frac{0}{b-a} = 0$$

1238

Then

$$\log_e T\Big]_1^2 = \log_e \frac{M^2\left(1+aM^2\right)^{\frac{a+1}{b-a}}}{\left(1+bM^2\right)^{\frac{b+1}{b-a}}}\Bigg]_1^2 \tag{A8}$$

By substituting the limits,

$$\log_e \frac{T_2}{T_1} = \log_e \left[\frac{\dfrac{M_2^{\,2}\left(1+aM_2^{\,2}\right)^{\frac{a+1}{b-a}}}{\left(1+bM_2^{\,2}\right)^{\frac{b+1}{b-a}}}}{\dfrac{M_1^{\,2}\left(1+aM_1^{\,2}\right)^{\frac{a+1}{b-a}}}{\left(1+bM_1^{\,2}\right)^{\frac{b+1}{b-a}}}}\right] \tag{A9}$$

Taking the antilog of equation (A9) and substituting for a and b gives

$$\frac{T_2}{T_1} = \frac{\dfrac{M_2^{\,2}\left(1 + \frac{\gamma-1}{2}M_2^{\,2}\right)^{\frac{\gamma+1}{\gamma+1+2\gamma K}}}{\left[1+\gamma(1+K)M_2^{\,2}\right]^{\frac{2(\gamma+1+\gamma K)}{\gamma+1+2\gamma K}}}}{\dfrac{M_1^{\,2}\left(1 + \frac{\gamma-1}{2}M_1^{\,2}\right)^{\frac{\gamma+1}{\gamma+1+2\gamma K}}}{\left[1+\gamma(1+K)M_1^{\,2}\right]^{\frac{2(\gamma+1+\gamma K)}{\gamma+1+2\gamma K}}}} \tag{A10}$$

Changing the fourth term on the right side of equation (A5) to

$$+ \frac{b^2}{a-b}\left(-\int_1^2 \frac{dM^2}{-1-bM^2} + \int_1^2 \frac{M^2dM^2}{-1-bM^2}\right)$$

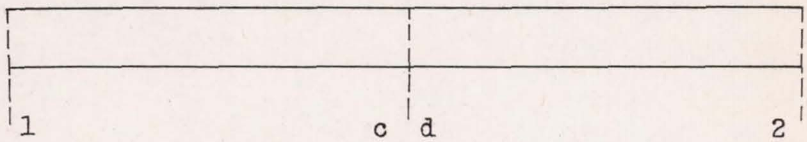
1238

and integrating in a similar manner gives a form of equation (A10) that can be evaluated for negative values of K.

APPENDIX B

SAMPLE CALCULATION

The pressure drop in an annular cooling passage will be calculated by the use of the large working chart in two steps. Given conditions:



Station	1	c d	2
	x = 0	2.50	5.00 feet
	T = 500	603	980° R

Mass velocity, G, lb/sec/sq ft	14.05
Inlet total pressure, P_1, lb/sq ft absolute.	1458
Hydraulic diameter, D_h, ft	0.0417
Average friction factor, $\bar{f}$.	0.00756

Inlet Mach number. - The inlet Mach number is obtained from the weight-flow parameter and figure 1.

$$\frac{G\sqrt{T_1}}{P_1} = \frac{14.05\sqrt{500}}{1458} = 0.2155$$

Then from figure 1, for $\gamma = 1.40$, $M_1 = 0.242$.

Intermediate Mach number. - The Mach number at station c is determined from the working chart by the following steps:

$$\frac{4\bar{f}}{D_h} = \frac{4 \times 0.00756}{0.0417} = 0.725$$

$$K_{1-c} = \frac{\frac{4\bar{f}l_{1-c}}{D_h}}{\log_e \frac{T_c}{T_1}} = \frac{0.725 \times 2.50}{\log_e \frac{603}{500}} = 9.67$$

Entering the working chart at $M = 0.242$ and $K = 9.67$ gives

$$\frac{4\bar{f}L^*_1}{D_h} = 5.20$$

which is solved for L^*_1:

$$L^*_1 = \frac{5.20}{0.725} = 7.17 \text{ feet}$$

$$L^*_c = L^*_1 - l_{1-c} = 7.17 - 2.50 = 4.67 \text{ feet}$$

Multiplication of L^*_c by $\frac{4\bar{f}}{D_h}$ gives

$$\frac{4\bar{f}L^*_c}{D_h} = 0.725 \times 4.67 = 3.38$$

Reentering the working chart at $\frac{4\bar{f}L^*_c}{D_h} = 3.38$ and $K = 9.67$, $M_c = 0.298 = M_d$.

Outlet Mach number. - The outlet Mach number is found in the same manner as the intermediate Mach number.

$$K_{d-2} = \frac{\frac{4\bar{f}l_{d-2}}{D_h}}{\log_e \frac{T_2}{T_d}} = \frac{0.725 \times 2.50}{\log_e \frac{980}{603}} = 3.73$$

Enter the working chart at $M = 0.298$ and $K = 3.73$ and read

$$\frac{4\bar{f}L^*_d}{D_h} = 2.18$$

Solve for L^*_d

$$L^*_d = \frac{2.18}{0.725} = 3.01 \text{ feet}$$

and

$$L^*_2 = L^*_d - l_{d-2} = 3.01 - 2.50 = 0.51 \text{ feet}$$

or

$$\frac{4\bar{f}L^*_2}{D_h} = 0.725 \times 0.51 = 0.37$$

Reentering the chart at $\frac{4\bar{f}L^*}{D_h} = 0.37$ and $K = 3.73$, $M_2 = 0.567$.

Pressure drop. - The total pressure at station 2 is obtained from the weight-flow parameter. The weight-flow parameter at station 2 is found from figure 1 for $\gamma = 1.40$ and $M = 0.567$,

$$\frac{G\sqrt{T_2}}{P_2} = 0.431$$

Solving for P_2 gives

$$P_2 = \frac{14.05\sqrt{980}}{0.431} = 1020 \text{ lb/sq ft}$$

The loss in total pressure across the cooling passage is

$$P_1 - P_2 = 1458 - 1020 = 438 \text{ lb/sq ft}$$

or if the loss in static pressure is desired

$$p_1 = P_1 \left(\frac{p_1}{P_1}\right)$$

$$p_2 = P_2 \left(\frac{p_2}{P_2}\right)$$

By use of reference 10,

$$p_1/P_1 = 0.9601$$

$$p_2/P_2 = 0.8040$$

$$p_1 = 1458 \times 0.9601 = 1400 \text{ lb/sq ft}$$

$$p_2 = 1020 \times 0.8040 = 820 \text{ lb/sq ft}$$

and the loss in static pressure is

$$p_1 - p_2 = 580 \text{ lb/sq ft}$$

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1238

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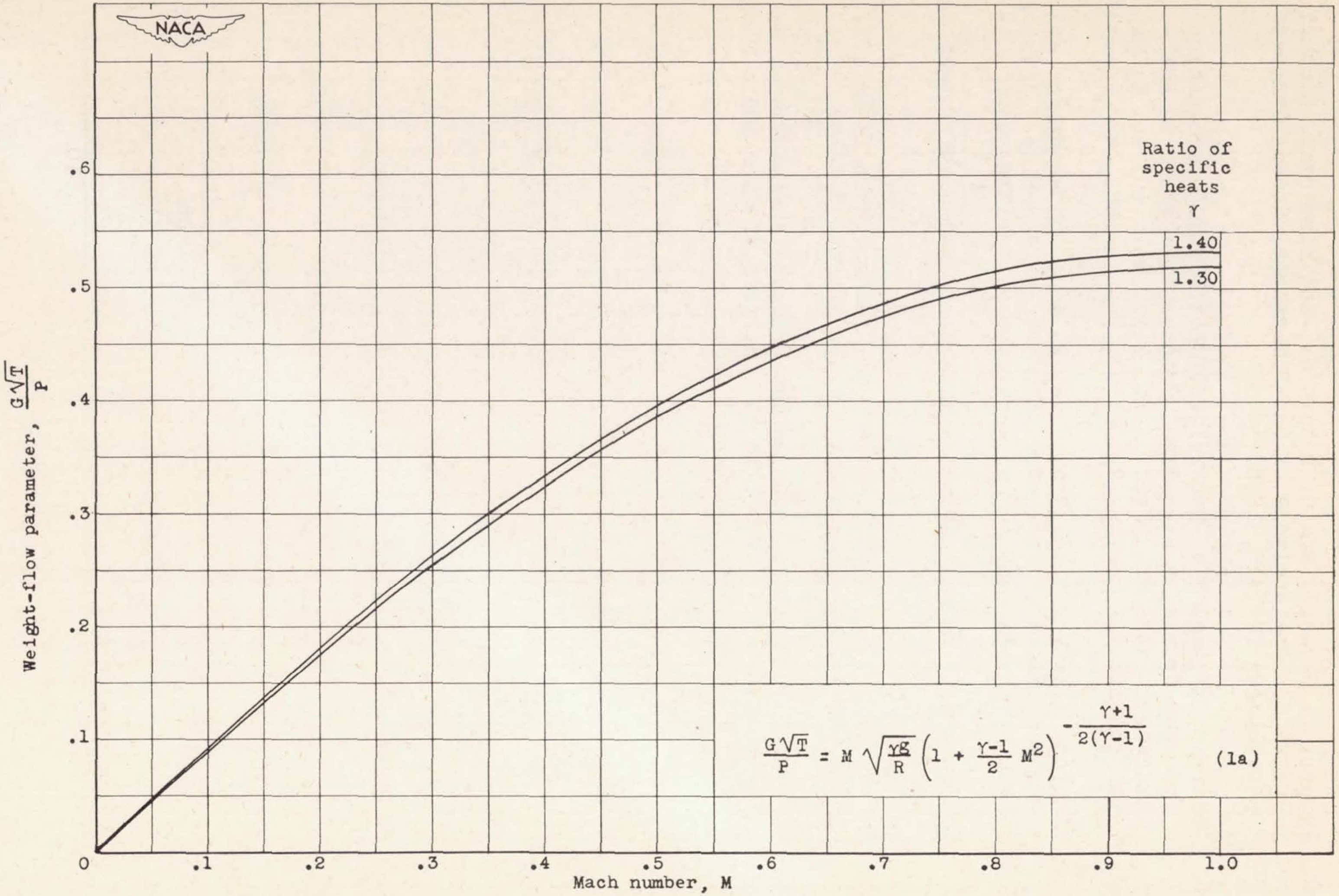


Figure 1. - Variation of weight-flow parameter $\frac{G\sqrt{T}}{P}$ with Mach number M.

1238

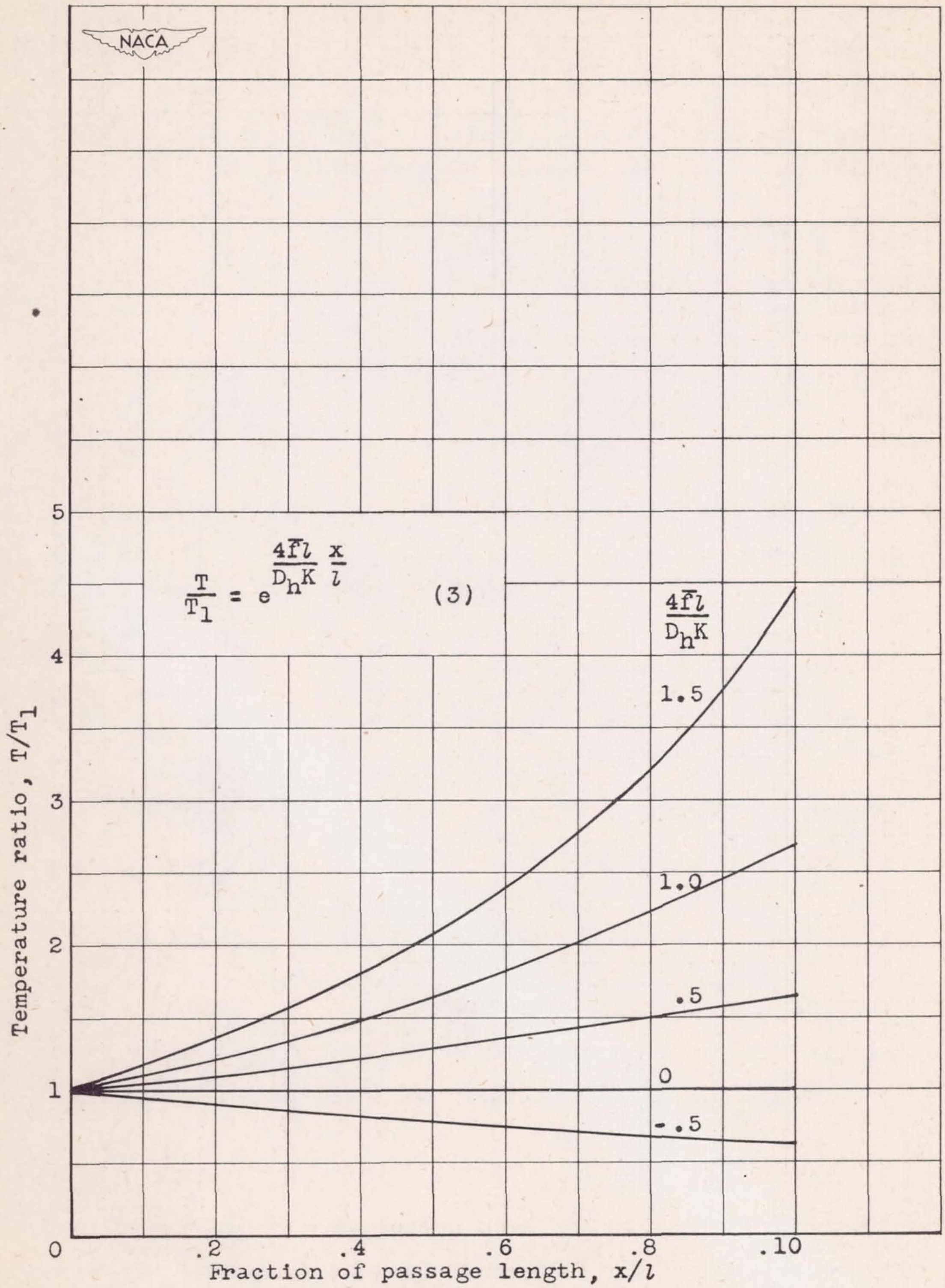


Figure 2. - Temperature profile assumed in integration of differential equation for simultaneous friction and heat transfer.

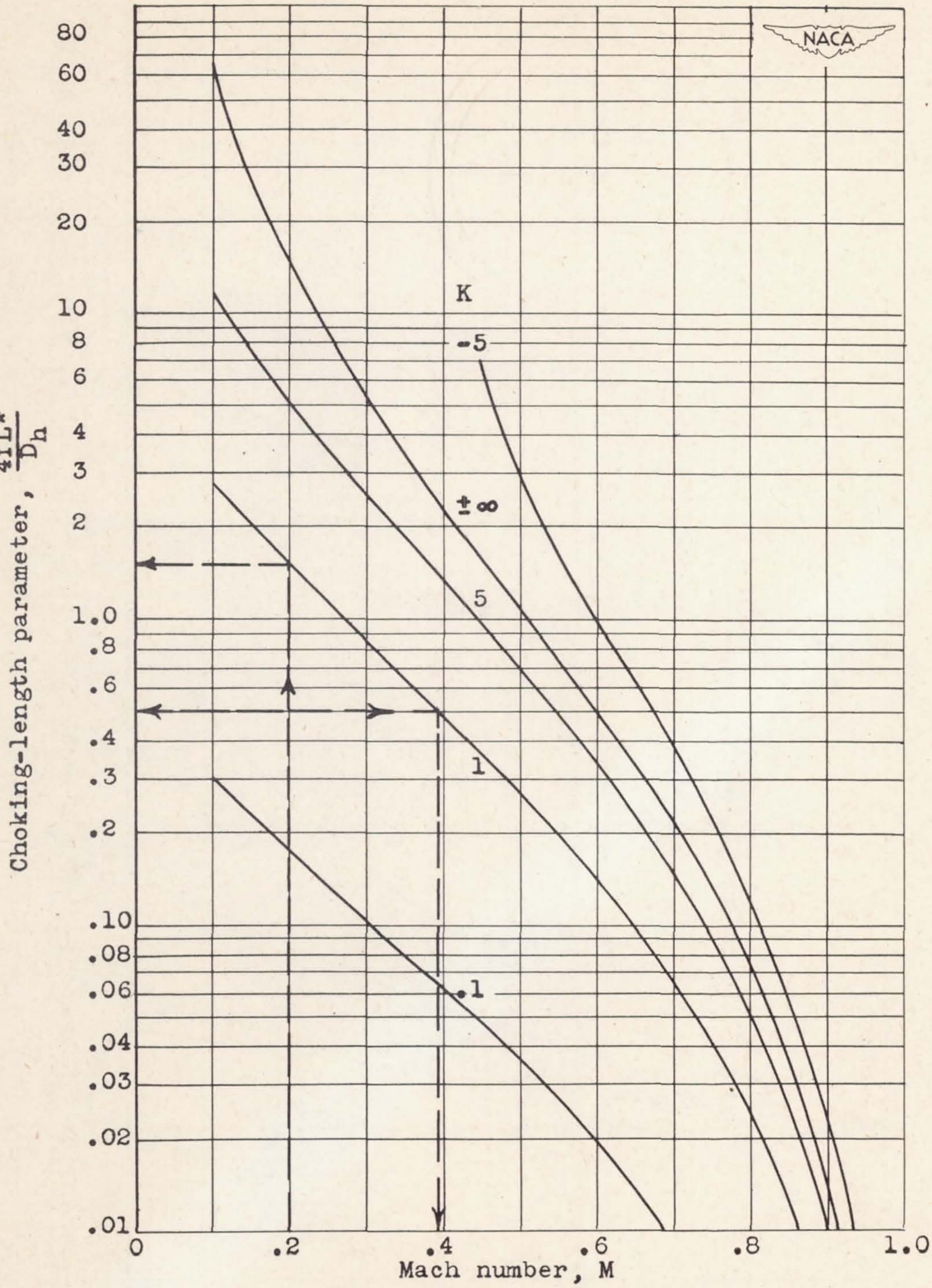


Figure 3. - Chart for determination of Mach-number variation in compressible fluid flowing through constant-area duct with simultaneous friction and heat transfer. Ratio of specific heats γ, 1.40. (A 16- by 16-in. working chart of this figure is enclosed.)

1238

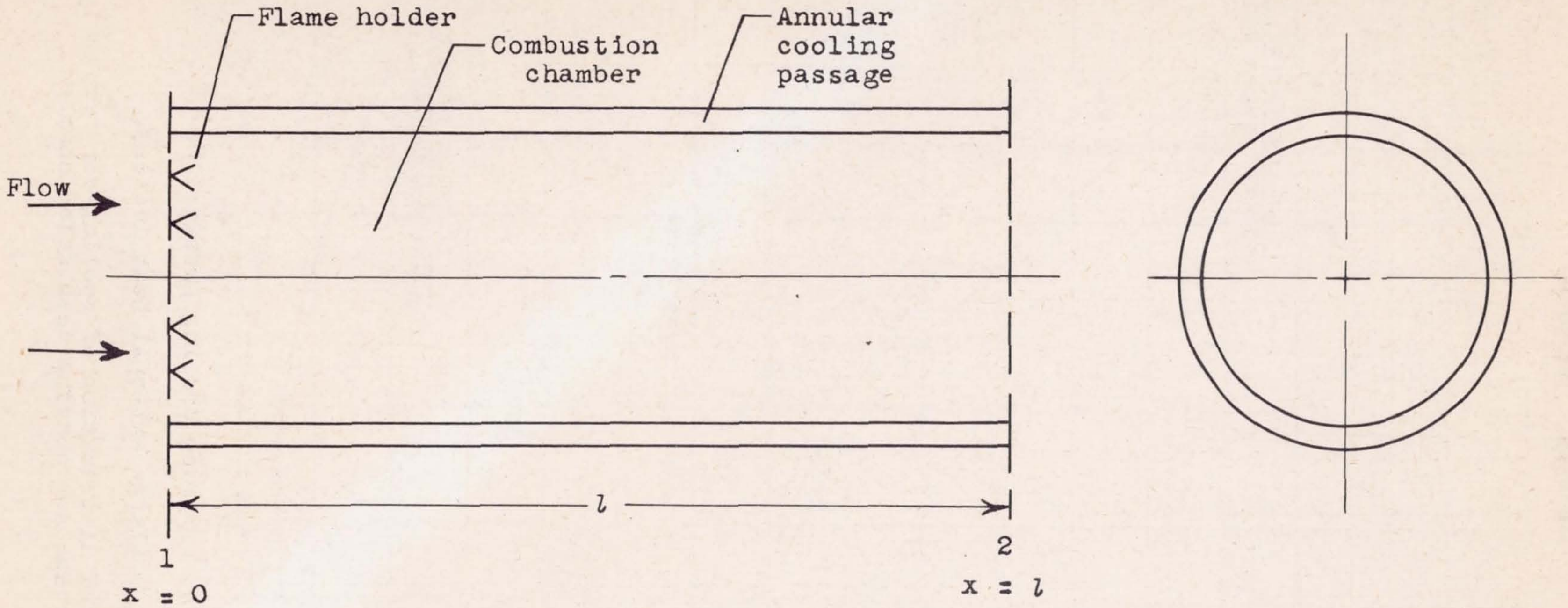
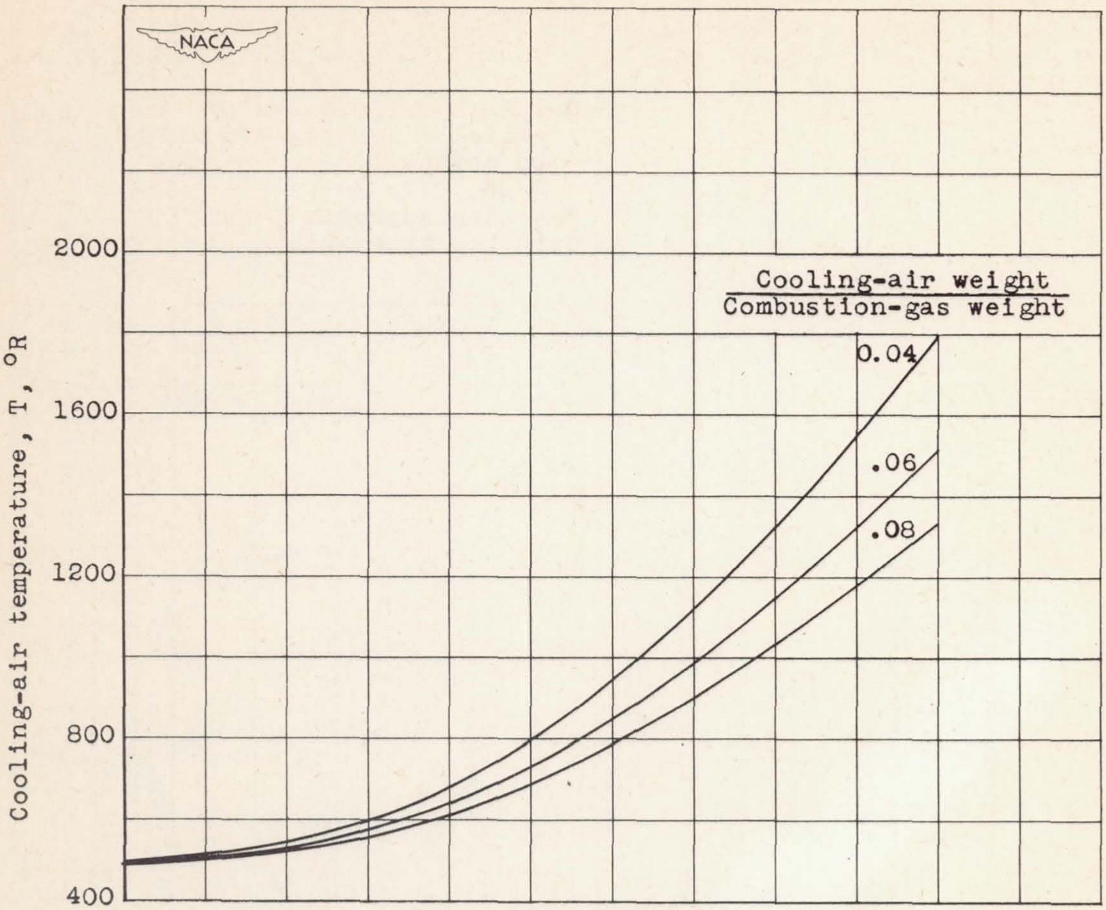
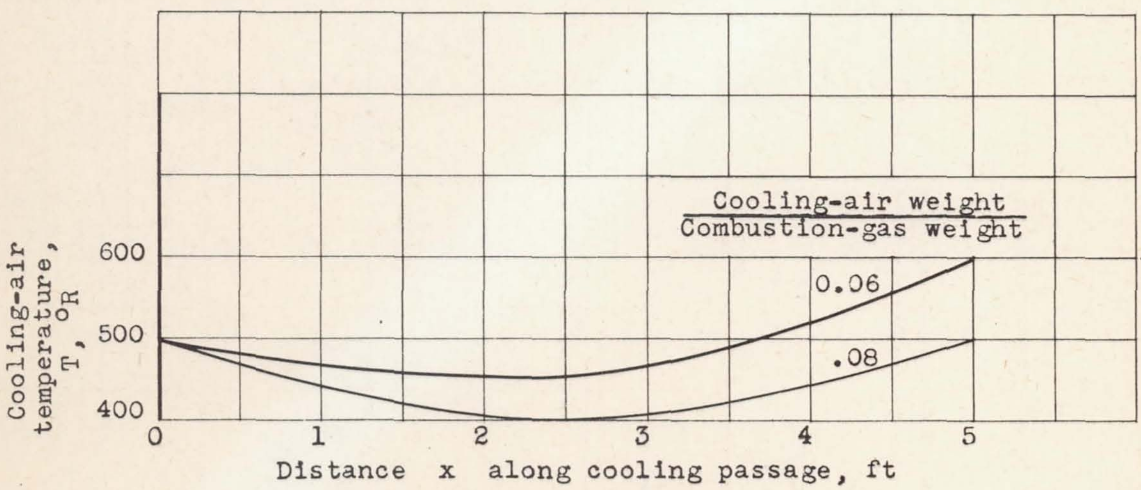


Figure 4. - Schematic diagram of combustion chamber cooled by air flowing through annular passage.



(a) Case A (no external heat losses).



(b) Case B (large external heat losses).

Figure 5. - Longitudinal distribution of cooling-air temperature in annular passage surrounding combustion chamber.

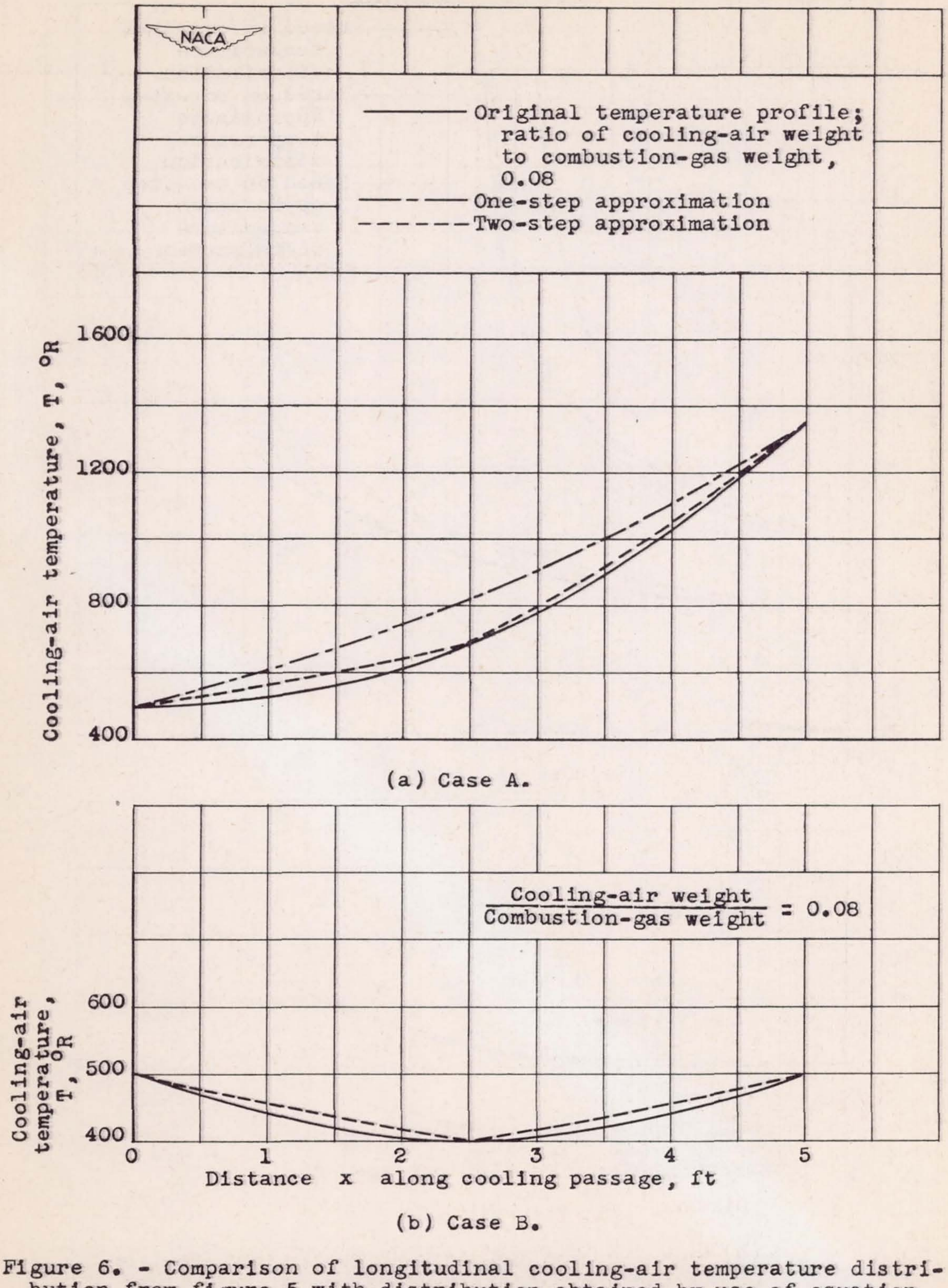


Figure 6. - Comparison of longitudinal cooling-air temperature distribution from figure 5 with distribution obtained by use of equation (3).

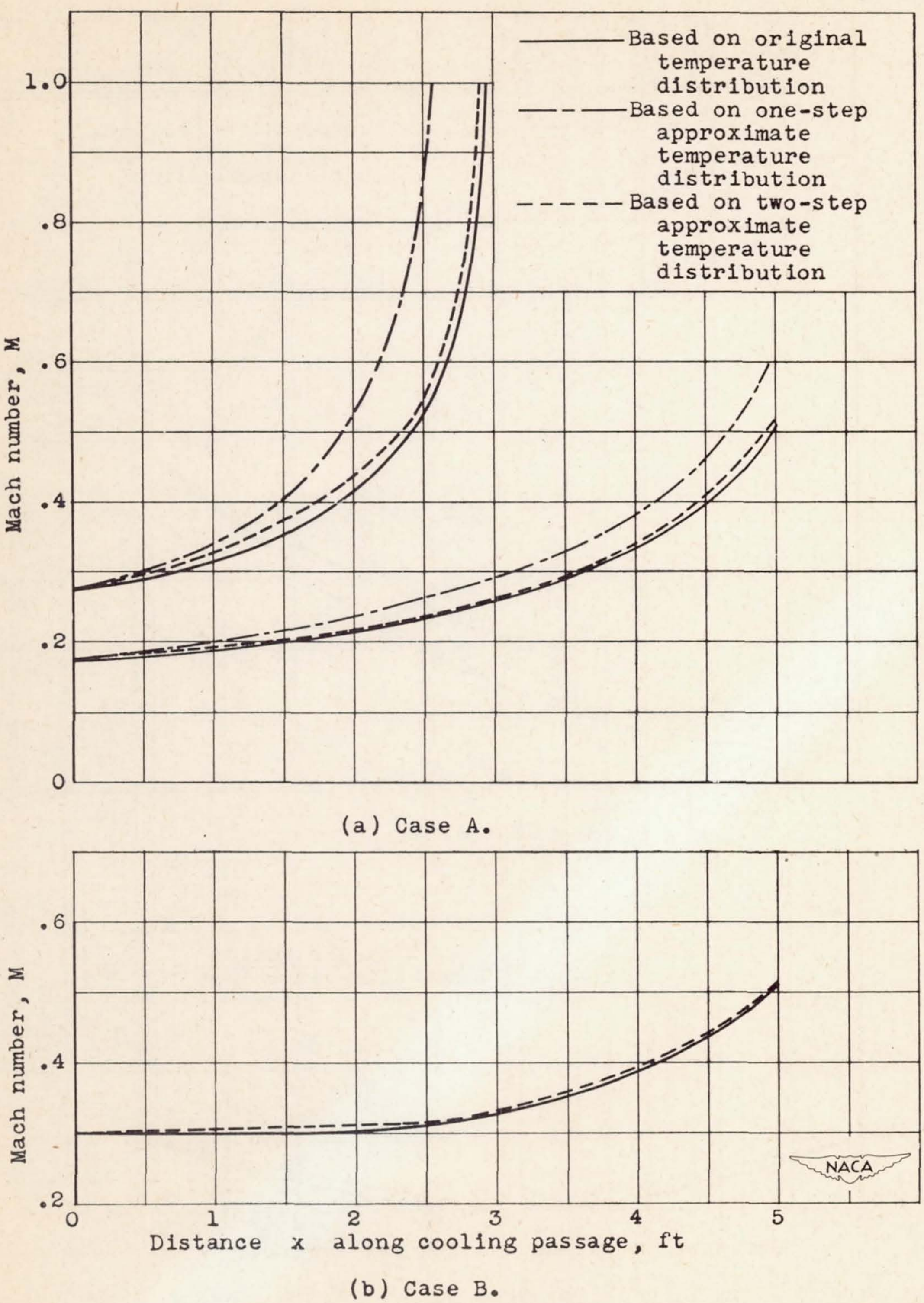


Figure 7. - Comparison of cooling-air Mach number distribution based on original and approximate temperature distributions of figure 6.

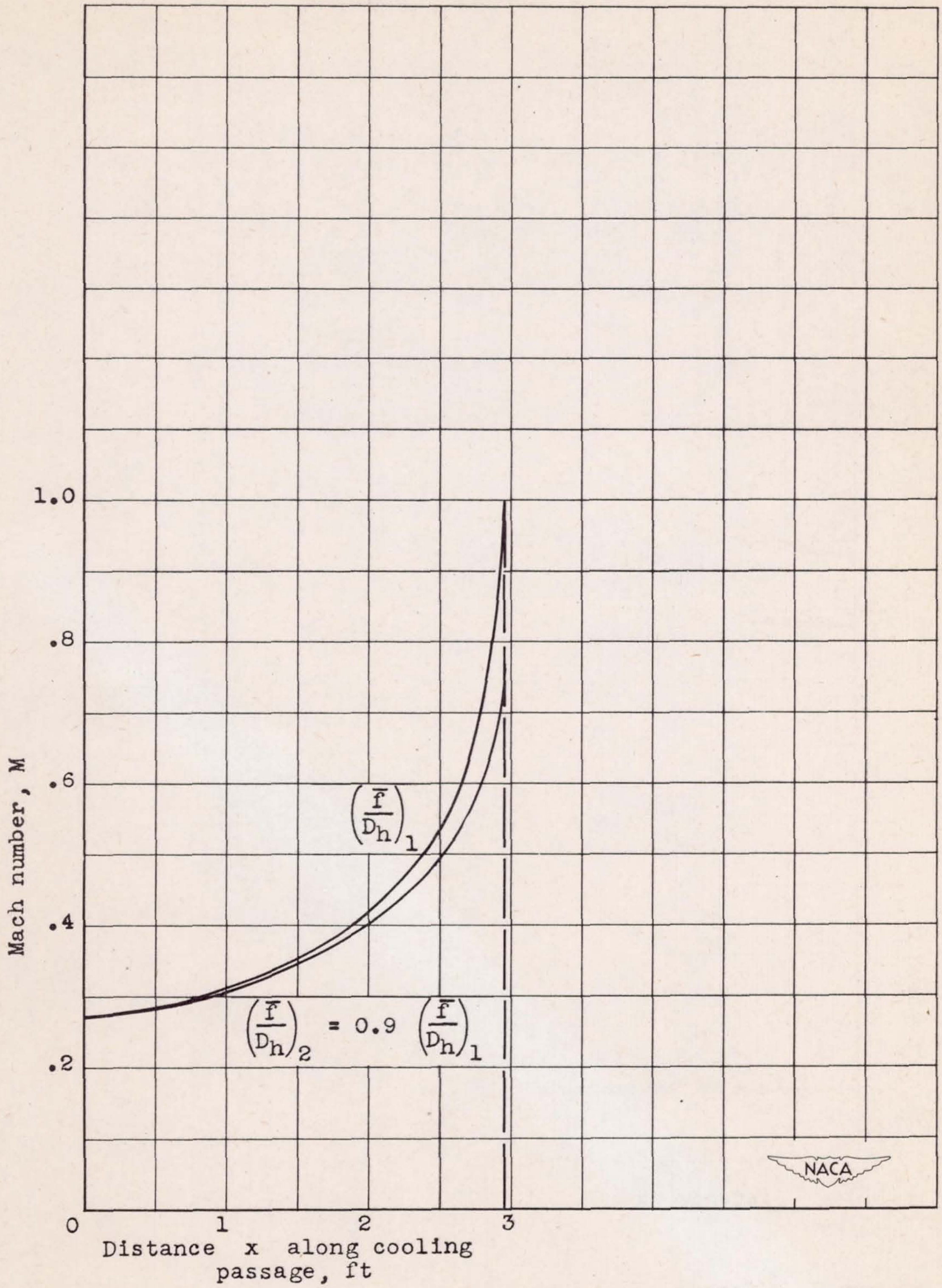


Figure 8. - Effect of $\bar{f}/D_h$ on Mach number profile.

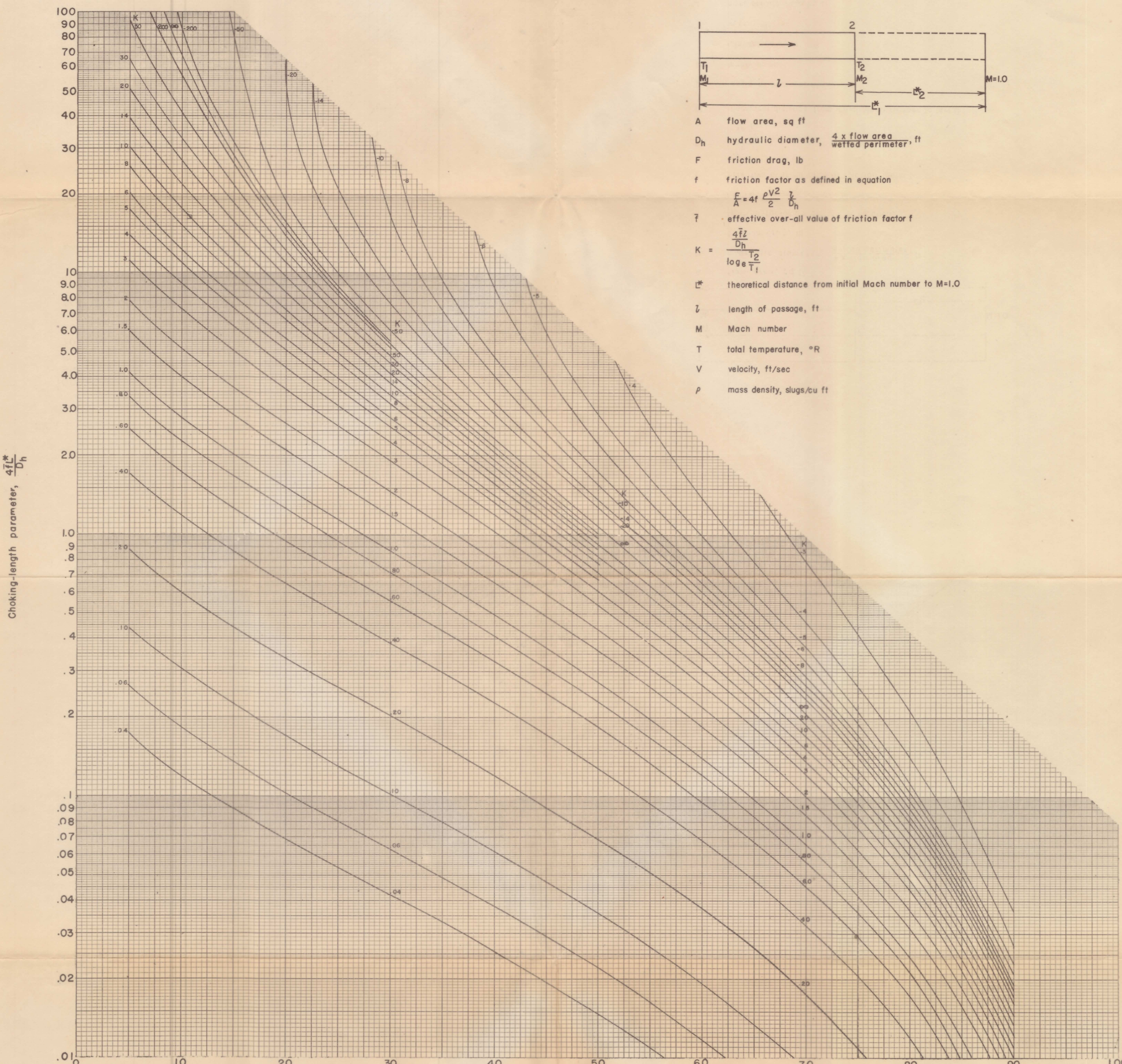


Figure 3.- Chart for determination of Mach-number variation in compressible fluid flowing through constant-area duct with simultaneous friction and heat transfer. Ratio of specific heats, γ, 1.40.